On the adequacy of qualifying Roger Penrose as a complex Pythagorean

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Abstract

The aim of the presented article is to provide an in-depth analysis of the adequacy of designating Penrose as a complex Pythagorean in view of his much more common designation as a Platonist. Firstly, the original doctrine of the Pythagoreans will be briefly surveyed with the special emphasis on the relation between the doctrine of this school and the teachings of the late Platonic School as well as its further modifications. These modifications serve as the prototype of the contemporary claims of the mathematicity of the Universe. Secondly, two lines of Penrose's arguments in support of his unique position on the ontology of the mathematical structures will be presented: (1) their existence independent of the physical world in the atemporal Platonic realm of pure mathematics and (2) the mathematical structures as the patterns governing the workings of the physical Universe. In the third step, a separate line of arguments will be surveyed that Penrose advances in support of the thesis that the complex numbers seem to suit these patterns with exceptional adequacy. Finally, the appropriateness of designation Penrose as a complex Pythagorean will be assessed with the special emphasis on the suddle threshold between his unique position and that of the adherents of the mathematicity of the Universe.

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1. Introduction

The renowned British mathematical physicist, Roger Penrose, belongs to the most recognized adherents of the mathematical platonism. In the most general terms, mathematical platonism refers to an array of beliefs that mathematical objects are independent of the human cognition and that they exist extramentally in an abstract and atemporal world of mathematical ideas (e.g. Wójtowicz, 2002, pp. 20-24). Consequently, scientific inquiry in the area of mathematics does not lead to the *construction* of these objects but to their *discovery*. In his original and critically acclaimed book entitled *The Emperor's New Mind* (Penrose, 1989), Penrose presented his philosophical views taking eventually the form of the ontology of the three worlds: *mathematics*, *physics* and *mind* (Penrose, 1994, pp. 411–421; 2005, pp. 7–24, 1027–1033). It must be clearly emphasized that Penrose by no means regards this ontology as the ultimate solution but rather as *three mysteries* that will not cease to reveal their intricacies. In particular, his interpretation of the Gödel theorem and the hypothesis of the coupling of consciousness with the quantum gravitational effects provoked massive opposition of experts from a broad variety of disciplines (Penrose, 1994, pp. 64–116; for an extensive review see e.g. Grygiel and Hohol, 2009).

Since platonism prevails as the ontological stance among mathematicians and theoretical physicists, Penrose's views on the ontological status of mathematical objects do not provoke any marked dispute. His platonism, however, bears a very specific mark for Penrose passionately singles out the complex numbers as having a privileged ontological importance in comparison to other abstract mathematical structures. For this very reason, Mateusz Hohol named him "a complex Pythagorean" to reflect Penrose's deep belief in the reality of the complex numbers as "the stuff of the Universe" (Hohol, 2009). Such a designation seems all the more justified keeping in mind the utterance of Bertrand Russell who stated that "perhaps the oddest thing about modern science is its return to Pythagoreanism" (Ghyka, 2001, p. 311). Moreover, the tendency to refer to the Pythagorean doctrine as proper to the ontological status of the theoretical entities in the context of the contemporary physical theories manifests itself through the appearance of such philosophical positions as *neopythagoreism* (Krajewski, 2011).

The aim of the presented article is to expand on Hohol preliminary and somewhat provocative suggestion and provide an in-depth analysis of the adequacy of designating Penrose as a complex Pythagorean in view of his much more common designation as a Platonist. Some elements of this analysis have been already presented by Grygiel and constitute the proper point of departure of this study (Grygiel, 2014, pp. 267–278). Firstly, the original doctrine of the Pythagoreans will be briefly surveyed with the special emphasis on the relation between the doctrine of this school and the teachings of the late Platonic School as well as its further modifications proposed by Plato's pupils: Speusippus and Xenocrates. These modifications serve as the prototype of the contemporary claims of the mathematicity of the Universe. Secondly, two lines of Penrose's arguments in support of his unique position on the ontology of the mathematical structures will be presented: (1) their existence independent of the physical world in the atemporal Platonic realm of pure mathematics and (2) the mathematical structures as the patterns governing the workings of the physical Universe. In the third step, a separate line of arguments will be surveyed that Penrose advances in support of the thesis that the complex numbers seem to suit these patterns with exceptional adequacy. Finally, the appropriateness of designation Penrose as a complex Pythagorean will be assessed with the special emphasis on the suddle threshold between his unique position and that of the adherents of the mathematicity of the Universe.

2. Pythagoras and Plato on mathematics

The attempts to locate the ontological views of the contemporary mathematicians and theoretical physicists such as Roger Penrose within the framework of the thought of the philosophers of the ancient Greece face several difficulties (e.g. Śleziński, 1999, pp. 206–228). Firstly, the contemporary scientists have a quite different point of departure in their inquiry for they directly handle highly abstract mathematical structures in contrast to the Greek mathematics of simple numbers applicable to the observed phenomena. Although there seems to be much propensity to juxtaposing the contemporary mathematicians and physicists with such ancient thinkers as Pythagoras, Plato and Aristotle, this propensity does not find uniform support. Paul Pritchard expresses his dissatisfaction as follows (Pritchard, 1995, p. 177):

> More particularly, the so called ‘Platonist’ philosophy (or philosophies) of (modern) mathematics owe nothing to Plato except for the spurious respectability derived from attaching his name to a set of views that he never held, and of which, could he understand them, he would be unlikely to approve.

Secondly, the original views of these thinkers in regards to the ontological status of mathematics were often subject to considerable modifications within their own schools (e.g., Plato). Moreover, the contemporary interpretations of these views often diverge quite considerably thereby making the respective comparisons and qualifications all the more problematic.

Not unlike many other ancient schools of thought, the members of the Pythagorean School strived to explain the harmony of the Universe's complexity as composed of elements remaining in opposition with respect to each other. Their unique solution came through music as they recognized that musical harmonies are achieved by strings of the instruments having their lengths as ratios of the natural numbers. The main point of contention between the existing interpretations of the Pythagorean doctrine is whether the natural numbers do indeed constitute the fundamental ontology or they are just the means of description of the regularities in nature. The ontologically strong interpretation prevails especially in the standard textbooks of the history of philosophy and seems to be the one that enjoys the common acceptance (Tatarkiewicz, 1970, pp. 41–48, Copleston, 1994, pp. 29–37). On this interpretation, it is maintained that these are indeed the natural numbers that are the essence of the harmony of the Universe and that numbers have spatial dimensions whereby the ratios of the physical lengths express the corresponding harmonies.

However, recent in-depth comparative studies carried out by Dembiński show that the ontological strength of this interpretation has to be considerably weakened (Dembiński, 2015). He claims that this interpretation has its source in Aristotle's expositions of the philosophical doctrines of the antique thinkers preceding him that bear the bias of his own views. According to Dembiński, the Pythagoreans did not equate mathematics with ontology but for them the *arché* of the Universe was the harmony resulting from the interaction of the two highest existential principles: the Limit (*peras*) and the Unlimited (*apeiron*)[1]. The role of mathematics is descriptive only in that it provides means to capture the regularities and patterns of the phenomena observed in nature. These means, that is the forms and shapes that can be assigned to phenomena, are called *eide*. In other words, the means by which the human mind has epistemic access to the structure of the Universe cannot be matched with the Universe's ontology. At this point it remains beyond doubt that Hohol's attempt to designate Penrose as a complex Pythagorean is contingent is upon the ontologically strong interpretation of the Pythagorean views on the nature of mathematics and careful conceptual distinctions and qualifications will have to be made to verify the adequacy of this designation.

Further insight into the adequacy of designating Penrose as a complex Pythagorean is obtained as one takes into account the modification of the Pythagorean views on mathematics introduced by Plato. As Dembiński points out (Dembiński, 1997, 2003), Plato's intervention involved the separation of the mathematical forms from the patterns and regularities occurring in nature with the subsequent elevation of these patterns and regularities to the status of the principles of the cosmic order, namely, to that of the ideas. Consequently, they fundamentally differ from the mathematical objects which are located below them in the hierarchy of being and are the constructs of the power of the human intellect bearing the name of *dianoia* (Dem-

[1] In order to substantiate his views on the ontology of mathematics of the Pythagorean School Dembiński refers to the commentary of Gajda-Krynicka (Gajda-Krynicka, 2007, pp.65-73; 151-193).

biński, 2001). The mathematical objects assume all the limitations proper to the human condition and they cannot constitute the ontological foundation of anything that exists (Dembiński, 2015).

In its late phase, the Platonic School was influenced by the Pythagoreans resulting in a reformulation of the Platonic understanding of the nature of mathematics. This change was mainly stimulated by the pupils of Plato: Speusippus (410-339), Xenocrates (396-314) and Eudoxos (408-355) who made mathematics the principal subject of discussions in the academy. Eventually, these discussions gave rise to the belief that it is mathematics that constitutes the fundamental fabric of the Universe. The contribution of Speusippus involves the assignment of all the characteristics of the ideal numbers to the mathematical numbers, that is, the separate existence, eternity, unchangeability and objectivity (Dembiński, 2010, pp.109-138). Xenocrates can be properly credited with turning mathematics into ontology (Dembiński, 2010, pp. 139–170). According to Dembiński, Xenocrates deserves to be named the precursor of the stance of the *mathematicity of the Universe* adhered to by many of the contemporary philosophers of physics and philosophers of science (Dembiński, 2010, p. 158). On this view, the mathematical structures do indeed underpin the fundamental fabric of the Universe (e.g. Heller, 2010; Życiński, 2013). As Życiński clearly points out, the mathematicity of the Universe has not yet developed into a uniform and well justified doctrine and, despite of its seemingly widespread popularity, still faces conceptual difficulties (Życiński, 2010). With this concise exposition of the Pythagorean doctrine in hand, a detailed critical analysis of the proximity of Penrose's ontological views to those of the Pythagoreans can be now undertaken.

3. Objectivity of mathematics

In his approach to the ontological status of mathematics, the greatest concern of Penrose is associated with the *objectivity of mathematics*, that is, its independence of the workings of the human mind or any social or cultural consensus. First and foremost, Penrose exploits the contrast between the unreliability of the human mind in its judgments and the precision of the scientific theories and claims that the reason for this precision must lie in the extramental objective reality (Penrose, 2005, p. 12). Interestingly enough, Penrose's choice of platonism does not originate from his particular fascination with the thought of Plato or the Greek philosophy in general but from the adequacy of the Platonic ontology to accommodate the basic observation: "It tells us to be careful to distinguish the precise mathematical entities from the approximations that we see around us in the world of physical things" (Penrose, 2005, p. 12). In order to give this idea its proper expression, Penrose coins out the term *robustness* to indicate the rigidity and unchangeability of mathematics that forces the human mind to accept its standards without much room for any creativity on the side of a mathematician. Penrose attests to this stance in the following way:

> Mathematics itself indeed seems to have a robustness that goes far beyond what any individual mathematician is capable of perceiving. Those who work in this subject, whether they are actively engaged in the mathematical research or just using results that have been obtained by others, usually feel that they are merely explorers in the world that lies far beyond themselves – a world which possesses objectivity that transcends mere opinion, be that opinion their own or the surmise of others, no matter how expert those others might be (Penrose, 2005, p. 13).

Penrose's observations are consonant with the opinions of other mathematical physicists such as Connes (Changeux and Connes, 1995) and Heller (2006, pp. 78–79). In support of this line of argumentation, Penrose refers to the specificity of the mathematical proof in which, as he insists, once a given mathematical statement is proven to be true, its truthfulness does not depend on the opinion or consensus of the community of mathematicians. Although Penrose clearly acknowledges the subjective aspect of the *acceptance* of a given proof, which is treated as valid once the community of mathematicians finds it convincing (e.g. Davis and Hersh, 1981, pp. 39–40; Harel and Sowder, 2007), he regards each proven theorem as objectively true and belonging to the Platonic world regardless of any attempt of demonstration. In justifying this inference, Penrose might be running into circularity for he evidently lacks here an independent condition allowing for an access to mathematical truth. This condition becomes available through Penrose's controversial interpretation of the Gödel theorems (Penrose, 1989, pp. 99–148; 2005, pp. 374–378).

As the next source of support for the objectivity of mathematics Penrose indicates the richness of the mathematical structures despite of their extremely simple definition. The Mandelbrot set is most frequently quoted by him to show that the intrinsic structure of this set is so rich that it could not possibly be the invention or design of any human mind (Penrose, 1989, pp. 92–98; 1994 2005, pp. 15–17). The objectivity of the set becomes manifest when its complicated and elaborate structure reveals itself as unchangeably the same regardless of any scientific method aimed at deciphering its nature. This means that the structural details as well as those of any mathematical structures are timeless and their only fitting mode of existence is in the atemporal Platonic world of mathematical forms. This argument is additionally corroborated by the fact that most of the mathematical structures that have been carefully investigated by the mathematicians do not find their physical application (Penrose, 2005, p. 18).

With the issue of the objectivity of mathematics being addressed, Penrose develops his line of argumentation to substantiate the thesis that the entire physical world is governed according to mathematical laws. Although he calls it his prejudice, the entire voluminous work entitled *The Road to Reality* is devoted to the justification of this tenet. Penrose's argumentation rests on two fundamental pillars: (1) the extraordinary precision of the scientific theories formulated in the language of mathematics and (2) the dependence of the precision of physical theories on the sophistication of the mathematical formalism used. Penrose states the following (Penrose, 2005, pp. 27–28):

> An important point to be made about these physical theories (general relativity, quantum electrodynamics, and the more general gauge theories describing the operation of the strong and weak forces of particle physics) is that they are not just enormously precise, but depend upon mathematics of very considerable sophistication. It would be a mistake to think of the role of mathematics in basic physical theory as simply of an organizational character, where the entities which constitute the world just behave in one way of another, and our theories represent merely our attempts (sometimes very successful, nevertheless) to make some kind of sense of what is going on around us.

It seems quite obvious that in the excerpt just quoted Penrose intimates his fundamental disagreement with the instrumentalism and antirealism of Hawking (e.g. Hawking and Penrose, 1996, pp. 3–4). More importantly, however, Penrose makes ample use of a his favorite category pertaining to the specificity of the mathematical structures, namely, that of *sophistication*. This notion seems somewhat intuitive and fuzzy at the first glance but Penrose equips it with meaning that more precisely relates it to the properties of mathematical formalisms. He explicitly states that the first sixteen chapters of *The Road to Reality* are devoted to demonstrate the essence of mathematical sophistication (Penrose, 2005, p. xix). A careful glance through these chapters easily leads to an inference that a given mathematical structure is sophisticated when it exhibits a marked inner complexity, ordering and richness.

The fundamental connection between mathematics itself and its function in the governance of the workings of the Universe comes from the fact that the successful, that is, the empirically adequate theories involve formalisms with the high degree of sophistication. The increasing complexity of a given physical problem demands mathematical structures of greater sophistication for the problem's proper description leading to a concomitant increase in the precision of a given theory (Penrose, 1997, pp. 50–52). The comparison of the Newtonian dynamics with the general theory of relativity yields a fitting example in this regard where the elaborate tools of the differential geometry needed to be employed to reflect the complexity of physics of the gravitational field represented by the Einstein's field equation. Much greater sophistication of mathematical structures is anticipated in the future theory of quantum gravity (Penrose, 2005, pp. 958–1009).

4. The complex sophistication and the holomorphic philosophy

Penrose's understanding of the sophistication of mathematical structures is more precisely revealed in his account of the nature of the complex numbers (structures) which, as it has been indicated in the introduction to this study, are considered by him as the primary fabric of the Universe. First and foremost, Penrose uses this designation in regards to the mathematical structure of his main research focus in quantum gravity and lifetime project, namely, the *twistor theory* (Penrose, 1967). The main idea that Penrose articulates concerning the complex structures is that the results that require arduous computations with the use of the of the real structures are obtained "for free" as the complex structures come into play. As it has been the case with the Mandelbrot set, sophistication of a mathematical structure means that a simple formula unveils an extraordinary richness of structure covering a wide variety of detailed applications. Interestingly enough, this idea seems to be related to the Einsteinian demand of the simplicity as a guide to the choice of a good theory (Einstein, autobiographical notes). After all, in his controversy with Hawking, Penrose has been always likened to Einstein while Hawking to Bohr (Hawking and Penrose, 1996, pp. vii, 134–135). The adequacy of this match has been confirmed with some qualifications in a detailed analysis carried out by (Grygiel, 2014, pp. 328–336). However, the assessment to what degree Penrose's sophistication matches Einstein's simplicity would require a separate detailed conceptual study.

Before one delves into Penrose's survey of the properties of the complex numbers as presented in *The Road to Reality*, it is worthwhile to signal a considerable level of his infatuation with them resulting in statements as suggestive and rhetorical as ‘the magic of the complex numbers’. Penrose does not hesitate to push this magic to the extreme as he openly states:

> Nature herself is as impressed by the scope and the consistency of the complex-number system as we ourselves, and has entrusted to these numbers the precise operations of her world at its minutest scales (Penrose, 2005, p. 73).

As the first illustration of this magic Penrose brings forth the well known example of the solutions of the polynomial equations which are soluble in set of the complex numbers following the introduction of the imaginary factor *i*. This is the *fundamental theorem of algebra* (Penrose, 2005, p. 75). The “for free” strategy is clear: a simple formula unveils the richness of its internal structure yielding a whole array of possible solutions unobtainable with the use of the real numbers.

The most important testimony of the ‘magic of the complex numbers’, however, appears in the complex number analysis in regards to the differentiation of the complex functions (Penrose, 2005, pp. 122–134). In particular this concerns one of the mathematical concepts especially celebrated by Penrose, namely, that of a *holomorphic function*. Physicists particularly appreciate the so called *smooth functions* which are differentiable unlimited number of times and form a class of real functions denoted as C^∞. The reason for this is that most of the laws describing the dynamics of the physical systems appear in the form of the differential equations. The existence of the higher order derivatives assures that any quantities expressed as rates of changes of other quantities will retain their physical meaning regardless of the order of the derivative.

It turns out that the notion of smoothness can be enhanced by the so called *analyticity* of a function. A function is analytic at a given point p if it can be expressed as a Taylor series in a neighborhood of p. Analyticity improves smoothness in that it makes "gluing" together two different analytic functions. The class of "smoother" analytic functions is denoted as C^ω. The real gain in smoothness, however, occurs for the complex functions that fulfill certain conditions of regularity and the Cauchy-Riemann equations. Such functions do have a complex derivative and they bear the name of the *holomorphic functions*. In contradistinction to the real functions, the existence of the first complex derivative implies the existence of the derivatives of all higher orders "for free" and the analyticity of a given function (Penrose, 2005, pp. 122–123). Further manifestations of the sophistication of the holomorphic functions involve the property of the *analytic continuation*.

Penrose's particular focus on the sophisticated complex structures involves also a marked geometrical aspect. In general, geometrical representations of the mathematical structures constitute an essential element of his intellectual discipline. This brings onto the scene another key mathematical concept, namely, that of *conformal transformations* which Penrose meticulously illustrates with the Escher diagrams (Penrose, 2005, pp. 33–37).In a nutshell, conformal transformations are such that preserve the angles and their orientations. It turns out that holomorphic functions can be represented as conformal transformations. Penrose considers the *Riemann sphere* as the fundamental geometrical object to represent the holomorphic structures (Penrose, 2005, pp. 142–152). This sphere is a complex projective plane CP^1, the simplest of the compact Riemann surfaces. Penrose uses the Riemann sphere to illustrate two other manifestations of the sophistication (and 'magic') of the complex structures: the Fourier series and the hyper-functions. In order to additionally substantiate the unique properties of these structures Penrose compares them with *quaternions* to conclude that quaternions do not exhibit any of the sophistication comparable to that of the complex structures (Penrose, 2005, pp. 193 sqq). The demonstration of the uniquely sophisticated character of the complex structures yields an *a priori* argument on the privileged mathematical character of these structures referred to by Penrose as the *holomorphic philosophy* (Penrose, 2005, pp. 994, 1034, 1056).

5. In harmony with nature

Parallel to this, Penrose undertakes the effort to show that the complex structures do indeed seem to accord in an exceptional way with the regularities according to which the Universe operates. As the first instance of such harmony Penrose singles out quantum mechanics in which a quantum state is given by a linear superposition of eigenstates of a quantum operator with the complex coefficients. Their complex character is fundamental for the occurrence of the quantum effects. In particular, for a simple two spin particle, its state is given by a linear combination of two spin eigenstates. This situation can be easily mapped on a Riemann sphere which represents all possible states of the particle (Penrose, 2005, pp. 553–559). In giving this situation a geometrical interpretation, Penrose makes a remarkable comment by asserting that the representation the quantum states on this sphere offers a unique connection between the mathematical properties of these states and the ordinary directions in space. What seemed abstract so far, is now given a sense of tangibility. This inference will turn out to be relevant for the assessment of the adequacy of designating Penrose as a complex Pythagorean.

The next confirmation of the suitability of the complex structures to fit the constitution of the physical reality comes from the special relativity theory. In particular, these structures harmonize greatly with the transformation properties of the Minkowski space-time, namely, the Lorentz symmetry group. Penrose begins his clarification of this issue to by firstly referring to a visual experience of two observes who register the patterns of the stellar constellations (Penrose, 2005, pp. 428–431). The patterns registered by the observer at rest and the observer in motion correlate with each other via the conformal Möbius map. The conformal maps in general form a group of automorphisms preserving the complex structure of the Riemann sphere. Consequently, this sphere maps the Lorentz group of the special relativity and all the light rays crossing at a given point in space-time are equivalent to the Riemann sphere. The observer's field of vision, that is the celestial sphere, naturally maps onto the Riemann sphere thereby demonstrating its deep ties with the complex structures that are believed to constitute the stuff of the Universe. The possibility of a direct spatial representation of these structures seems to reach its most clear expression as he comments on the patterns of stars registered by observers that move with respect to each other (Penrose, 2005, pp. 430):

> This suggests a convenient labeling of the stars in the sky might be to assign a complex number to each (allowing also ∞)! I am not aware that such a proposal has been taken up in astronomy, but the use of such a complex parameter, called a 'stereographic coordinate', related to standard spherical polar angles by the formula $\zeta = e^{i\varphi} ctg(\frac{1}{2}\Theta)$ in general relativity theory.

Furthermore, the harmony of the complex structures with the physical reality becomes manifest in the quantum field theory which arises from the unification of quantum mechanics and the special relativity. The Riemann sphere represents a suitable mathematical object to represent the splitting of a complex periodic function into the negative and positive frequency components as holomorphic extensions into the northern and southern hemispheres (Penrose, 2005, pp. 161–164). This unique property allows for assigning a precise geometrical sense to the positive energy condition that is central for the stability of a given quantum system (Penrose, 2005, pp. 612–613). Also, this geometrical representation facilitates the distinction between the particle creation and particle annihilation processes and enhances the clarity of the physical interpretation of the antiparticles (Penrose, 2005, pp. 663).

The argument of the harmony of the complex structures with the physical reality is most applicable in support of the *twistor theory* as the theoretical framework with which Penrose wishes to achieve the unification of the quantum and gravitational phenomena (Penrose, 2005, pp. 958–1009). Due to the extremely abstract and sophisticated character of the structures involved (e.g., *cohomologies*), only a simple example of this harmony will be mentioned. A concise account of all other more sophisticated examples attesting to the harmony of the twistor spaces with the workings of the Universe has been provided elsewhere by Grygiel (2014, p. 265). In regards to the simple example of the spacetime of special relativity as represented with a twistor space Penrose concludes the following:

> Indeed, the fact that the Riemann sphere plays an important role as the celestial sphere in relativity theory requires spacetime to be 4-dimensional and Lorentzian, in stark contrast with the underlying ideas of string theory and other Kaluza-Klein-type schemes. The full complex magic of twistor theory proper is very specific to the 4-dimensional spacetime geom-

> etry of ordinary (special) relativity theory, and does not have the same close relationship to the 'spacetime geometry' of higher dimensions (Penrose, 2005, pp. 967).

6. A Platonist or a Pythagorean?

The detailed survey of Penrose's argumentation on the status of mathematics and its role in shaping the fabric of the Universe has confirmed that he subscribes to a separate mode of objective existence of the mathematical structures in the atemporal Platonic world of mathematical forms and in parallel to the physical world where mathematics incarnated as complex structures refracts the patterns of the nature's behavior. In addition to this, however, Penrose claims that the world of mathematics is the most primitive, that is, ontologically most fundamental in regards to the world of physics and to the world of the mental as usually depicted in his scheme of the three worlds – three mysteries: "mathematics is a kind of necessity virtually conjuring its very self into existence through logic alone" (Penrose, 2005, pp. 1029). Since the mathematical structures are the *raison d'être* of the physical structures, that is, physics is contingent upon mathematics in its existence and operation, Penrose's position in this regard seems to reflect that of Speusippus. On this view, mathematical structures constitute the source of harmony of the different disparate entities comprising the stuff of the Universe and not the "stuff", that is, the ontology itself. At this point it appears rational to suggest that Penrose is not a committed supporter of the mathematicity of the Universe in the strong ontological sense for he clearly states: "I might baulk at actually attempting to identify physical reality within the abstract reality of the Plato's world" (Penrose, 2005, pp. 1029) or “though I have strong sympathy with this idea of actually identifying these two worlds, there must be more to the issue than just that” (Penrose, 1989, p. 430). Penrose always refers to mathematical structures as rather providing a pattern according to which nature operates and not being the fabric that nature is made out of. Consequently, in his ontological views Penrose does seem to follow neither in the footsteps of Xenocrates nor the contemporary adherents of the mathematicity of the Universe. Although just qualified as a dualist “Speussipian”, Penrose evidently blurs this dualism by pointing to a smooth transition between the world of mathematics and physics (Penrose, 1997, p. 3):

> The more we understand about the physical world, and the deeper we probe into the laws of nature, the more it seems as though the physical world almost evaporates and we are left only with mathematics.

By looking at the above quote one is undoubtedly left with the question: what is the stuff of the Universe? What is that “evaporates” as one leaves the realm of physicality? These questions are left unanswered by Penrose but he does every right to do so as he has explicitly warned of his proposals being rather mysteries and conjectures than proven theorems. Be that as it may, at this point it is worthwhile asking how much of a Pythagorean Penrose really appears to be. The final verdict will certainly depend on the interpretation of pythagoreanism assumed. As is has been already signaled above, Penrose’s designation as a complex Pythagorean echoes most likely the more prevalent textbook interpretation claiming the ontology of the natural numbers combined with his great emphasis on the role of the complex numbers in the account of the workings of the Universe. In view of the newer interpretation of the Pythagorean doctrine put forward by Dembiński, however, the designation thus conceived fails on two counts because neither the Pythagoreans nor Penrose qualify as adherents of the mathematicity of the Universe. As a result, it bears only a rhetorical value and might be actually misleading by labeling Penrose with positions he would never subscribe to.

The adequacy of designating Penrose as a complex Pythagorean slightly improves as one reconsiders it in line with Dembiński's rendition of the Pythagorean doctrine. On this reading, mathematics pertains only to the patterns or regularities observed in nature and not to its underlying ontology. And this is indeed as far as Penrose's pythagoreanism can be advanced for while for the Pythagoreans mathematics plays only a descriptive role, for Penrose mathematics constitutes the ontic foundation of the patterns according to which the Universe operates and justifies the extraordinary precision of these operations. Evidently, the improvement is not great but at at least it singles one aspect in which to designate Penrose as a complex Pythagorean is not a mere figure of speech. However, taking into account rather low significance of this aspect in comparison to the dominating Platonic component in Penrose's ontology, this designation should be considered as of very little hermeneutic value.

Since Penrose articulates his philosophical positions from the point of view of a contemporary theoretical physicist, his perspective involves an immensely greater insight into the workings of the Universe achieved by means the modern scientific method. In particular, this regards experimentation leading to the discovery of patterns governing levels of physical reality below the surface of the unaided sensorial observation. This concerns both the levels of the atomic phenomena as well as those at the large scale of the Universe where abstract mathematical structures, e.g., the complex structures, have to be used to for the description of these phenomena. As metaphysical objects, however, these structures do not come into the direct contact with the human sensorial apparatus meaning that they could manifest themselves to this apparatus only through a suitable representation. Penrose achieves this effect with the use of the Riemann sphere which allows for the spatial representation of the abstract complex structures. Since this is merely a representation that is sensorially perceived, the knowledge of these structures is not exhaustive but is obtained through certain likeness only. Consequently, it is necessary to impose an additional the adequacy of the designation of Roger Penrose as a complex Pythagorean with the qualification that, unlike the natural numbers for the Pythagoreans, the complex structures can be sensorially accessed indirectly as *mediated* through the representation on the Riemann sphere. Such a modification of the Pythagorean stance seems to be inevitable in regards to all contemporary physical theories due to the highly abstract character of their mathematical formalisms.

Based on the nature of arguments in favor of the privileged status of the complex structures one could have a justified impression that Penrose turns this argumentation into ideology. As it was clearly demonstrated, he does not abstain from the use of such rhetorical formulations as the holomorphic philosophy suggesting that the complex numbers indeed might constitute a metaphysical necessity. Despite of this persuasive character of his discourse which definitely enriches his literary style, Penrose is far too well versed both in science in philosophy not to realize that his being ostensibly infatuation with the magic of the complex numbers is mainly rhetorical and nostalgic. The articulated ontological thesis bears the status of a mere hypothesis which he deeply believed (and most likely still does) to be conceptually promising in the search for the yet unknown theory of quantum gravity. The following quote clearly attests to Penrose's correct understanding of the relation between the context of discovery and the context of justification with the holomorphic philosophy evidently belonging to the first (Hawking and Penrose, 1996, p. 119):

> From the point of view of the complex-holomorphic ideology of twistor theory, a big bang with $k < 0$, leading to an open universe, is to be preferred (Stephen prefers a closed one). The reason is that only in a $k < 0$ universe is the symmetry group of the initial singularity a holomorphic group, namely just the Möbius group of holomorphic self-transformations of the Riemann sphere CP^1 (the restricted Lorentz group). This is the same group that twistor theory off in the first place – so for twistor-ideological reasons, I certainly prefer $k < 0$. Since this is based only on ideology I can, of course, withdraw it in future if I am wrong and the universe is, in fact, found to be closed!

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