

# Does the quantum mechanical wave function exist?

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Abstract

I address the question whether the wave function in quantum theory exists as a real (ontic) quantity or not. For this purpose, I discuss the essentials of the quantum formalism and emphasize the central role of the superposition principle. I then explain the measurement problem and discuss the process of decoherence. Finally, I address the special features that the quantization of gravity brings into the game. From all of this I conclude that the wave function really exists, that is, it is a real (ontic) feature of Nature.

Keywords

wave function, quantum mechanics, ontology, quantum formalism, superposition principle, measurement problem, decoherence, quantum gravity.

## 1. Quantum theory

The title of my contribution may sound somewhat surprising, at least at first glance. After all, the quantum mechanical wave function and its generalizations in quantum field theory (generically here called  $\Psi$ ) are standard tools in quantum theory and its many applications in physics, chemistry, and even biology. This is true, and

one can definitely say that  $\Psi$  exists in a mathematical sense. The question addressed here instead refers to whether  $\Psi$  can be attributed an ontic or merely an epistemic role, that is, whether  $\Psi$  can be attributed reality in the same way as, for example, an electric field possesses, or whether it merely describes something like an information catalogue (as Schrödinger once put it). This is a question that has occupied physicists since the advent of quantum theory in the 1920s and that still occupies them today; see, for example, (d'Espagnat, 1995; Kiefer, 2015a), or (2018) and the many references quoted therein. Here, I will argue that the answer to the question posed in the title is definitely in the affirmative, and I will try to put together the main arguments of why this is so and why the wave function has an ontic (real) status. Some of these arguments have been presented in an earlier article (Kiefer, 2012a), to which I will occasionally refer.

At the heart of all of quantum theory is the *superposition principle*. It can be separated into a kinematical and a dynamical version (Joos, Zeh et al., 2003). The kinematical version expresses the fact that if  $\Psi_1$  and  $\Psi_2$  are physical states, then  $\alpha\Psi_1 + \beta\Psi_2$  is again a physical state, where  $\alpha$  and  $\beta$  are complex numbers. For more than one degree of freedom, this leads to the important concept of *entanglement* (*Verschränkung*) between systems (Kiefer, 2015a), which plays a particular important role in modern developments such as quantum information. The very concept of a quantum computer relies on entanglement.

It is clear that this kinematical version only makes sense if it is consistent with the dynamics of the theory. But this is the case. The fundamental equation is the Schrödinger equation (by which I include its field theoretic generalization, the functional Schrödinger equation), and this equation is *linear*: the sum of two solutions is again a solution. An importance consequence of the superposition

principle is obvious: the space of what we may call “classical states” form only a tiny subset in the space of all possible states. A simple example is the superposition of two localized states, each of which can describe a classical state, to a nonlocal (and thus nonclassical) state. It must be emphasized that the quantum mechanical wave functions are not defined on spacetime, but on *configuration space* (cf. e.g. Zeh, 2016 for a lucid conceptual discussion). Except for the case of one particle, this is a high-dimensional space: the dimension is  $3N$  for a system of  $N$  particles, and infinite in field theory. Otherwise, there would be no entanglement between systems.

Entanglement is the central distinguishing feature of quantum theory. As already Erwin Schrödinger (1935, p. 555) put it:

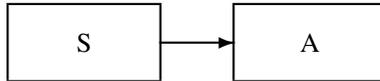
I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or  $\psi$ -functions) have become entangled. . . . Another way of expressing the peculiar situation is: the best possible knowledge of a *whole* does not necessarily include the best possible knowledge of all its *parts*, even though they may be entirely separated. . .

The superposition principle has been experimentally tested in uncountably many experiments (Schlosshauer, 2007; Kiefer, 2015a). Even before the term entanglement was coined, it was clear that the electrons in a helium atom must be entangled in order to lead to the correct observed binding energies (Hylleraas, 1929). Modern experiments include the interference of biomolecules, the entanglement of photon pairs over distances of hundreds of kilometres, and the observation of neutrino oscillations, to name only a few; see, for example, (Deng et al., 2019) for an experiment involving interference between light sources separated by 150 million kilometres. There can thus be

no doubt that the superposition principle holds. The generation of “macroscopic” superpositions is being seriously considered; see, for example, (Clarke and Vanner, 2018).

In the mathematical language of quantum theory, the validity of the superposition principle is encapsulated in the use of vector spaces for the quantum states (wave functions). The stronger concept of a Hilbert space (using a scalar product between states) is motivated by the probability interpretation of quantum theory, which by itself is connected to the “measurement problem” discussed since the early days of the theory. This measurement problem refers, in fact, to the only class of situations in which the superposition principle seems to break down.

What is the measurement problem? Let us consider the simple situation of an apparatus, A, coupled to a system, S:<sup>1</sup> I emphasize that



both system and apparatus are described by quantum theory. This analysis goes back to John von Neumann (1932). The simplest situation of an interaction is the “ideal measurement”: the system is not disturbed by the apparatus, but the state of the apparatus becomes correlated with the state of the system. If S is in a state  $|n\rangle$  and A in an initially uncorrelated state  $|\Phi_0\rangle$ , the total state of S and A evolves as

$$|n\rangle|\Phi_0\rangle \xrightarrow{t} |n\rangle|\Phi_n(t)\rangle. \quad (1)$$

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<sup>1</sup> This and the following diagramme are taken from our monograph (Joos, Zeh et al., 2003).

The measurement problem appears when we consider a superposition of possible states  $|n\rangle$ .<sup>2</sup> This leads to the evolution:

$$\left(\sum_n c_n |n\rangle\right) |\Phi_0\rangle \xrightarrow{t} \sum_n c_n |n\rangle |\Phi_n(t)\rangle, \quad (2)$$

where  $|\Phi_n(t)\rangle$  is the resulting state ('pointer state') of A. But (2) is a *macroscopic* superposition! Since such superpositions<sup>3</sup> are not observed, John von Neumann postulated the occurrence of a "collapse of the wave function" in measurement-like interactions; he did not, however, present a dynamical equation for such a collapse, which must be unitarity violating and is thus in conflict with the Schrödinger equation.

In more recent years, various models of wave function collapse have been presented in the literature and possible experimental tests have been discussed.<sup>4</sup> It must be emphasized that most of these models only make sense if the wave function acquires a real (ontic) status. This is different from its role in the Copenhagen interpretation of quantum theory, where the 'collapse' has the mere formal meaning of an information increase. We shall see in the next section how we can proceed without assuming a dynamical collapse, that is, without violating the unitarity of quantum theory.

Before doing so, I want to conclude this section with some remarks on relativistic quantum theory, in particular the Dirac equation. The Dirac spinor appearing there should not be confused with the wave function discussed above. The spinor is *not* defined in configuration space; it is defined on a classical (in general four-dimensional)

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<sup>2</sup> In the simple situation of spin- $1/2$ , one would have two states  $|n\rangle$ , one corresponding to (say) spin up and the other to spin down.

<sup>3</sup> An especially impressive example is Schrödinger's cat.

<sup>4</sup> See, for example, (Bassi et al., 2013) for a comprehensive review.

spacetime. It thus cannot describe entanglement and can only serve to address one-particle situations; it can describe correctly the situation in the hydrogen atom, but cannot even be formulated for the helium atom.<sup>5</sup> Relativistic quantum theory is only consistent in the form of quantum *field* theory; the Dirac equation follows from quantum electrodynamics (QED) for the special case of one-particle excitations. When we talk here about the ontological status of  $\Psi$ , this refers in the general case of quantum field theory to wave *functionals*. These functionals are defined on the configuration space of all fields; in the case of QED, for example, this is the space of all vector potentials and charged Grassmann (anti-commuting) fields.

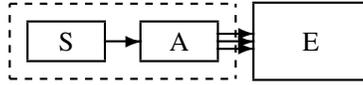
## 2. Decoherence

How can one understand the nonobservation of superpositions such as (2) without advocating an explicit collapse? The key role in answering this question is played by the presence of the ubiquitous environment for the apparatus. This was clearly recognized in the pioneering work by H.-Dieter Zeh in 1970.<sup>6</sup> ‘Environment’ is here a technical term that stands for additional degrees of freedom whose interaction with the ‘apparatus’ (or other systems under consideration) cannot be avoided. In concrete examples, these may be air molecules or photons that scatter off the ‘apparatus’. One thus has instead of the above diagramme the following situation:

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<sup>5</sup> Cf. in this context (Zeh, 2016).

<sup>6</sup> The original reference is (Zeh, 1970). See (Joos, Zeh et al., 2003) for details and references.



Here, E stands for the environmental degrees of freedom, and the three arrows between A and E indicate the many degrees of freedom.

If the environment is in an initial state  $|E_0\rangle$ , the superposition principle for the whole system of S, A, and E leads to

$$\left( \sum_n c_n |n\rangle |\Phi_n\rangle \right) |E_0\rangle \xrightarrow{t} \sum_n c_n |n\rangle |\Phi_n\rangle |E_n\rangle. \quad (3)$$

But this is an even more macroscopic superposition than (2) because it not only includes system and apparatus, but also the many degrees of freedom of the environment. Has the situation not become worse now? The answer is no. The reason is because the degrees of freedom of the environment are in general not accessible; when dealing with S and A only, one has to trace them out and to consider instead the reduced density matrix of S and A alone, from which all *local* observations follow. Since different environmental states are in general orthogonal (because they can discriminate between different states of A),  $\langle E_n | E_m \rangle \approx \delta_{nm}$ , the reduced density matrix assumes the form

$$\rho_{SA} \approx \sum_n |c_n|^2 |n\rangle \langle n| \otimes |\Phi_n\rangle \langle \Phi_n|, \quad (4)$$

which is approximately (but not identically) equal to a classical stochastic mixture. The information about the original superposition of (2) has now been transferred to a nonlocal correlation between S and A on the one side, and E on the other side. They are no longer observable at S and A itself: “The interferences exist, but they are not *there*.” The various system states  $|n\rangle$  are distributed with proba-

bilities  $|c_n|^2$  according to the Born rule of quantum theory. (It should be noted, though, that the very notion of density matrix is based on the validity of the Born rule.)

This irreversible emergence of classical properties (nonobservability of interference terms) through the unavoidable interaction with the environment is called *decoherence*. It has been explored in the last decades, both experimentally and theoretically.<sup>7</sup> According to decoherence, macroscopic objects *appear* classically, although they are fundamentally described by quantum theory. Decoherence is a process that can be treated entirely within standard quantum mechanics and which can be based on realistic processes discussed in a quantitative manner.<sup>8</sup>

What are the consequences of this for the interpretation of quantum theory in general and for the wave function in particular? If the superposition principle and the Schrödinger equation are universally valid, one arrives at what is called the Everett or many-worlds interpretation (see e.g. d’Espagnat, 1995; Zeh, 2016). Unitary quantum theory is then exact and never violated. The dead and the alive Schrödinger cat, for example, then indeed exist simultaneously in different “Everett branches”, and also the observer seeing the cat exists in two versions. In this point of view, the wave function definitely has an ontic status and exists in the way discussed above. The Everett interpretation together with decoherence makes the measurement problem obsolete.

A question often asked is about the derivation of the probability interpretation (the Born rule) in the Everett picture. This has been

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<sup>7</sup> Major reviews are Joos et al. (2003), Zurek (2003), Schlosshauer (2007). Crucial experiments are also discussed in Haroche (2014).

<sup>8</sup> Such quantitative calculations were presented for the first time in Joos and Zeh (1985).

discussed at length in the literature; see, for example, Zurek (2018) and the references therein. The probability interpretation only makes sense for situations in which decoherence is effective, because only then the various alternatives can be treated independently and probabilities can be assigned. Whether the Born rule can then really be *derived* or only be made plausible, is a contentious issue. But what is clear that the Everett interpretation together with decoherence and the Born rule gives a consistent picture that is not in need of completion.

The Everett interpretation is the simplest one at the level of the mathematical formalism. The fundamental equations are all linear. It is not a simple interpretation if one sticks to a classical picture of the world. This is what the main alternative—explicit collapse models—wants to achieve (see e.g. Bassi et al., 2013). But this requires a modification of the usual formalism by bringing in nonlinearities or stochastic terms. Also here, the wave function assumes an ontic status. The main task is to work out concrete models and to explore their experimental status.

A rather mild modification is the de Broglie–Bohm theory. The Schrödinger for  $\Psi$  is left untouched, but in addition particle (or classical field) configurations are introduced. The wave function, which is defined in configuration space, acts as a kind of ‘guiding field’ for the particles in ordinary space. There, too, it has an ontic status and can thus be assumed to exist. At least in nonrelativistic quantum mechanics, the predictions of the de Broglie–Bohm theory agree with the predictions of standard quantum theory.

The prototype of an epistemic point of view is the Copenhagen interpretation. There,  $\Psi$  merely provides an increase of information during a measurement and has no physical existence on its own—only the classical concepts such as particle positions have. But is

such a point of view really consistent and satisfactory? This is hard to believe. New light on these interpretational questions is shed by entering the realm of quantum gravity and quantum cosmology. This is the topic of my final section.

### 3. Quantum gravity

In 1957, a group of distinguished physicists met at the University of North Carolina to explore the prospects of gravitational physics. This also included the possible quantization of the gravitational field. In the discussion, Richard Feynman came up with the following *Gedankenexperiment*. In a Stern–Gerlach type of setting, a particle is brought into a superposition of, say, spin up and spin down. Introducing some interconnections to a macroscopic object, say a ball of 1 cm diameter, one can bring the ball into a superposition of being translated upwards and downwards. But this corresponds to a superposition of two measurable gravitational fields (measurable e.g. with a Cavendish balance). Feynman then states (De Witt, 1957):

... if you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment. ... It may turn out, since we've never done an experiment at this level, that it's not possible ... that there is something the matter with our quantum mechanics when we have too much *action* in the system, or too much mass—or something. But that is the only way I can see which would keep you from the necessity of quantizing the gravitational field. It's a way that I don't want to propose. ...

In other words, unless one assumes that the superposition principle and the standard formalism of quantum theory is violated when

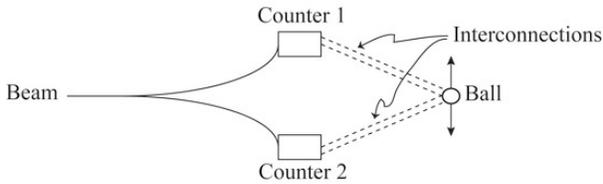


Figure 1: Feynman's *Gedankenexperiment* in which a microscopic superposition is transferred to a macroscopic one of a ball being simultaneously at two different places. Figure adapted from (De Witt, 1957).

gravitational fields play a role (as, for example, Lajos Diósi and Roger Penrose envisage), the quantization of gravity seems unavoidable. The majority of researchers thus accepts the assumption of extrapolating the standard linear formalism of quantum theory to quantum gravity. This holds for almost all of the existing approaches, from canonical and covariant quantum gravity up to string theory (Kiefer, 2012b).

At present, there is a discussion about the possibility to observe gravitational superpositions in the laboratory. There are proposals to probe a nonclassical gravitational field generated by two masses each of which is superposed at two locations (see e.g. Marletto and Vedral, 2017) or to probe such a field generated by the superposition of one mass in the spirit of Feynman's proposal cited above (see e.g. the remarks in Schmöle et al., 2016). The observability of such superpositions also meets with criticism (Anastopoulos and Hu, 2018).

What are the consequences of quantum gravity for our question about the reality of the wave function? In order to answer this question, it is sufficient to use the simplest and most conservative ap-

proaches to quantum gravity, which is quantum geometrodynamics.<sup>9</sup> One arrives at this theory when asking the following question: what is the quantum formalism that gives back Einstein's equations in the semiclassical (WKB) limit? This is analogous to the heuristic procedure that Erwin Schrödinger led to his equation in 1926.

The canonical formalism of general relativity discloses the real dynamical quantity of the theory: it is the *three-dimensional* geometry. The configuration space is thus the infinite-dimensional space of all three-dimensional metrics, with an additional constraint which guarantees that metrics related by coordinate transformations are counted only once. The theory possesses four local constraints, which after Dirac quantization are heuristically transformed into quantum constraints on physically allowed wave functionals. In a shorthand notation, they read

$$\hat{H}\Psi = 0, \quad (5)$$

where  $\hat{H}$  denotes the Hamilton operator of all gravitational and non-gravitational degrees of freedom. The functional  $\Psi$  is defined on the space of three-metrics and nongravitational fields. Equation (5) is also called Wheeler–DeWitt equation.<sup>10</sup>

One recognizes from (5) the absence of any external time parameter (see in this context Kiefer, 2015b). This is obvious for conceptual reasons. In classical relativity, spacetime (four-geometry) plays the same role that a particle trajectory plays in mechanics. After quantization, spacetime has disappeared in the same way as the particle trajectory has disappeared in quantum mechanics. But whereas in

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<sup>9</sup> Details and relevant references can be found, for example, in my monograph (Kiefer, 2012b).

<sup>10</sup> More precisely, if written out, (5) includes the Wheeler–DeWitt equation and the diffeomorphism constraints.

quantum mechanics Newton's absolute time  $t$  has survived, no such absolute time is present in Einstein's theory. As a result, the fundamental quantum gravity equations are timeless.

Of special concern here is quantum cosmology—the application of quantum theory to the Universe as a whole. In the simple case of a Friedmann universe with scale factor  $a$  and a conformally coupled scalar field  $\chi$ , the Wheeler–DeWitt equation assumes the form (after some rescaling and with a suitable choice of units):

$$\hat{H}_0\psi(a, \chi) \equiv (-H_a + H_\chi)\psi \equiv \left( \frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2 \right) \psi = 0. \quad (6)$$

How can one interpret such equations? At the most fundamental level, there is no time and there are no classical observers who could perform measurements. Therefore, the Copenhagen interpretation which requires the need of classical measurement agencies from the outset, is inapplicable. The question thus arises in which limit approximate notions of time and observers (more generally, of classical properties) emerge and what relevance this emergence has for the interpretation of the wave function.

Such a limit exists and is well understood (Kiefer, 2012b; 2015b). It is similar to the Born–Oppenheimer approximation in molecular physics. If one adds inhomogeneous degrees of freedom to the Hamiltonian in (6), the Wheeler–DeWitt equation is of the form

$$\left( H_0 + \sum_n H_n(a, \phi, x_n) \right) \Psi(a, \phi, \{x_n\}) = 0, \quad (7)$$

where the  $x_n$  stand for the inhomogeneities (gravitational waves, density perturbations). Writing  $\Psi = \psi_0 \prod_n \psi_n$  and assuming that  $\psi_0$  is

of WKB form, that is,  $\psi_0 \approx C \exp(iS_0/\hbar)$  (with a slowly varying prefactor  $C$ ), one gets

$$i\hbar \frac{\partial \psi_n}{\partial t} \approx H_n \psi_n \quad (8)$$

with

$$\frac{\partial}{\partial t} := \nabla S_0 \cdot \nabla.$$

This is nothing but a set of time-dependent Schrödinger equations for the inhomogeneities with respect to a time variable  $t$  that is defined from the homogeneous cosmological background;  $t$  is also called ‘WKB time’ and controls the dynamics in this approximation. Only in this limit can one talk about the probability interpretation of quantum theory and the existence of observers. It is thus not at all obvious whether the standard notion of Hilbert space need, or even can, be extrapolated to the level of full quantum gravity (beyond this level of the Born–Oppenheimer approximation).

In quantum cosmology, arbitrary superpositions of the gravitational field and matter states can occur. How can we understand the emergence of an (approximate) classical Universe? This is achieved by the process of decoherence introduced above (Kiefer, 2012a). Decoherence is a process in configuration space, and the irrelevant degrees of freedom can be taken to be part of the inhomogeneities  $x_n$ . In this way, the scale factor  $a$  and the field  $\chi$  can be shown to assume classical properties. The same then holds for WKB time  $t$ , which is constructed from these background variables. After this classicality is understood, one can investigate decoherence for some relevant inhomogeneous degrees of freedom; these include the inhomogeneities of an inflaton scalar field and of the metric, giving rise, after decoherence, to the observed CMB anisotropies and the (not yet discovered) primordial gravitational waves. In all these considerations, the wave

function is assumed to be real (ontic); this is also the case if one applies collapse models to quantum cosmology. I should also mention that even the problem of the arrow of time can, at least in principle, be understood in the framework of timeless quantum cosmology (Zeh, 2007).

It is clear that the debate about the correct interpretation of quantum theory will continue, at least until a clear experimental decision is reached (which could take quite a while). In this contribution, I have collected arguments which strongly support the point of view that the wave function is real (ontic), in the same way as, say, an electric field, is real. Thus, the wave function *exists*. The perhaps most important open question is: what is the configuration state for the wave functional at the most fundamental level? In canonical quantum gravity, it is the space of three-geometries plus nongravitational fields; what it is at the level of a fundamental quantum theory of all interactions, is unknown.

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