“Internal” Problems of Normative Theories of Thinking and Reasoning

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Abstract

This paper provides moderate criticism of so-called normative theories of thinking and reasoning. The discussion focuses on the problems of idealization, adequacy, inconsistent yet non-trivial logics, logical omniscience etc. I called them “internal” to the normative approach, because they stem from the very properties of formal systems used to model these two human activities. Some arguments, however, refer to the current theories in cognitive science, including those which are developed within “descriptive” framework.

Keywords

normative theories of thinking and reasoning, logic, probability theory, rational choice theory

1. Introduction

Normative theories of thinking and reasoning (later: NTTR) have been developing for centuries. Moreover, such theories have been a starting point for research on thinking and reasoning carried out within contemporary cognitive science. These theories provided the theoretical framework allowing to conceptualize or even operationalize the notions of thinking and reasoning. They have also been treated as a source of research hypotheses about the nature of these two processes. Nowadays, the normative approach is not a popular one. For example, in *Oxford Handbook of Thinking and Reasoning* published in 2013 only one out of forty chapters is devoted to normative theories. In contemporary literature they serve rather as a "strawman" or at least as a counterexample or suitable background for presenting theories of different kind (descriptive ones).

In this paper, I would like to present some problems besetting NTTR. In the presentation of these theories (section 2) I will follow the article of Chater and Oaksford (2012) published in the *Oxford Handbook*. Some ideas included therein will also appear in the last section of this study. Nevertheless, the main part of this paper is the discussion (section 3). Arguments presented in this paper concern problems resulting from the use of formal tools in normative approach. These problems arise from the very structure of normative theories and from the mode of explanation that such theories adopt. For this reason, these problems are called "internal".

2. Normative theories of thinking and reasoning

The basic difference between normative and descriptive theories of thinking and reasoning (later: DTTR) is the approach used to describe these processes. Normative theories try to determine how people *ought to* think or how reasoning *should* be carried out. On the other hand, the goal of descriptive theories of thinking and reasoning is to investigate a real nature of these processes – what they *really* are and how they are *actually* carried out. (In other words, descriptive theories aim to provide a *description* of thinking and reasoning.) While the former approach is most commonly associated with process of reasoning being modeled directly within certain formal system (e.g., logical calculus, probability theory or rational choice theory), the latter utilizes the *aparatus* of natural sciences – one builds a model, which is then subjected to the test of empirical research.

The main claim of Chater and Oaksford (2012) is that formal systems utilized in normative approaches impose on thinking the condition of consistency – logic demands the consistency of *beliefs*; probability theory imposes this requirement on the *degree* of beliefs; finally, rational choice theory requires our *choices* not to be contradictory.

2.1. Logic

Some scholars try to model thinking directly within a logical calculus, most often classical logic. The main condition imposed by classical logic onto a set of beliefs is that the set of beliefs should be consistent. Let us use the toy example – it seems to be incorrect to hold the following three beliefs simultaneously:

(1) All bullfrogs are sophisticated.
(2) Jeremiah is a bullfrog.
(3) It is not true that Jeremiah is sophisticated.

All bullfrogs are sophisticated, so Jeremiah is as well, since he is a bullfrog. Yet proposition (3) stands in contradiction with this conclusion. Logic provides formal language and strict methods of conducting such reasoning. Below the above reasoning is formalized in classical predicate calculus.

(1*) $\forall x(B(x) \rightarrow S(x))$
(2*) $B(a)$
(3*) $\neg S(a)$
(4) $B(a) \rightarrow S(a)$ $\quad$ $\forall$ elimination (1)
(5) $S(a)$ $\quad$ *Modus Ponens* (4),(2)
contradiction

Where $\forall$ means *for all*, $\rightarrow$ stands for *if..., then...*, variable x represents objects of the universe of discourse, whereas a is an individual name and stands in our case for Jeremiah. Predicate letters B and S mean respectively *is a bullfrog* and *is sophisticated*. Formula (4) is produced by the rule of universal quantifier elimination applied to formula (1). Formula (5) is produced by *Modus Ponens* applied to formulas (4) and (2). If someone holds above three beliefs, she has a pair of two contradictory sentences incorporated into set of her beliefs – $S(a)$ and $\neg S(a)$. We can define inconsistent systems in the fallowing way:

Defintion 1 *Let T be a deductive system whose language has a symbol for negation. T is said to be inconsistent if the set of its theorems contains at least two formulas or sentences, one of which is the negation of the other; otherwise T is consistent.*

In classical propositional calculus, a coherence condition is expressed by the law of (non)contradiction.

$$\neg(p\wedge\neg p)$$

But why (classical) logic does not allow contradiction? In inconsistent systems, i.e. in systems in which a pair of two contradictory formulas has been asserted, any proposition can be proven. Systems, in which anything can be proven are called trivial.

Defintion 2 *T is called trivial if the set of its formulas (or sentences) coincides with the set of its theorems; otherwise T is called non-trivial.*

The inference leading from contradiction to any proposition is captured by the law of explosion (other names: *ex contradictione quodlibet*, the principle of Duns Scotus).

$$p \wedge \neg p \rightarrow q$$

or

$$p \rightarrow (\neg p \rightarrow q),$$

where q is an arbitrary proposition. If we consider a logically closed set of believes containing two contradictory sentences, we have to admit that it contains also all sensible statements that are possible to express in given language.

2.2. Probability

The logic can be understood as a tool used to indicate that belief system is (not) contradictory. Nevertheless, the "all or nothing" strategy is often not applicable to the acquisition or keeping of a belief. Sometimes we keep a belief that we are not quite certain of, or acquire a belief from a source not fully dependable or trustworthy. Thus, we may assign some *degrees* to a belief. How to understand them? One can try to define the degree of a belief as the *probability* that this belief is true. Therefore, the proper formal calculus to explore the coherence of degrees of beliefs is probability theory. In cognitive science this approach is called Bayesian, since Thomas Bayes is the author of the most famous theorem concerning conditional probability of an event.

The basic version of Bayes’ Theorem follows directly from the definition of conditional probability

$Pr(A|B)$ is the probability that A is true under the condition that B is true.

The probability that both beliefs are true, $Pr(A,B)$ is equal to both the probability that B is true, $Pr(B)$ multiplied by the $Pr(A|B)$ and vice versa – the probability that A is true multiplied by $Pr(B|A)$.

$$Pr(A,B) = Pr(B)Pr(A|B) = Pr(A)Pr(B|A)$$

From this equation Bayes’ theorem can be obtained.

$$Pr(B|A) = \frac{Pr(A|B)Pr(B)}{Pr(A)}$$

Shortly speaking, Bayes’ theorem allows to calculate unknown probability from the known probabilities. It is often used in contemporary attempts to model knowledge in artificial intelligence, cognitive science or even linguistics.

2.3. Rational choice

From the point of view of the theories of thinking and reasoning logic can be understood as a framework imposing a condition of consistency on beliefs, while the probability theory imposes this condition on the degree of belief. To what extent can we impose similar condition on the choices we made? Proper formal framework that cope with this kind of questions is rational choice theory.

According to rational choice theory, rationality or irrationality of beliefs can not be assessed out of context. It may seem bizarre to choose presentation at the conference (P) over the holiday (H), but it is not irrational. Nevertheless, there is something odd in choosing P, when $\{P,H,D\}$ is offered, while choosing H from the set of options $\{P,H\}$ lacking the third "decoy" option. To exclude this pattern of behavior rational choice theory introduces *contraction* condition. Similarly, it seems unreasonable to choose P, when $\{P,H\}$ or $\{P,D\}$ is offered, while not choosing P from all three options $\{P,H,D\}$. The *expansion* condition is introduced to rule out this type of pattern.

If the above conditions are met, there is a preference relation "at least as good as" over the set of choices, which means that any choice is at least as good as any other items in the set of choices. If this preference relation is transitive (if X is at least as good as Y and Y is at least as good as Z, then X is at least as good as Z), then it is an ordering. This relation orders the set of our options with the one that will bring the least benefit at the beginning and most favored options at the end. This way we can formulate a simple rational choice criterion – from the set of options choose the one that will bring you most benefit.

Rational choice theory is still being developed. It may be made more complicated by adding the probability of obtaining benefits or imposing additional conditions on our choices. Then the criterion given above changes to the criterion of maximizing expected benefits, but the essence of the theory remains the same. Probably one of the most important extensions of rational choice theory is game theory, in which one’s choices depend on the choices of other participants of the game.

3. Discussion

Normative approach to modeling thinking and reasoning is a source of many problems. In the title of this work I called them “internal”, because they stem from the very properties of formal systems. Some arguments presented here refer to the results of cognitive science, including those which are based on descriptive theories of thinking and reasoning.

3.1. The problem of idealization

The first and obvious problem that normative theories face is a problem of excessive idealization, which is done during the attempts to formalize beliefs and reasoning. Formal systems utilize certain language and the price for the accuracy it offers is the loss of the enormous realm of content included in natural language (and even greater realm of content, to which we have access through what we call thinking).

3.2. The problem of adequacy

System of beliefs can be formalized using a number of different languages and the structure of reasoning may be expressed utilizing many different operators of logical consequence. In other words, there is no single logic, but there is a whole *continuum* of them. The sole use of classical logic (even in its predicate version) is not uncontroversial and nonclassical calculi are variform to the extend that it rises the problem of adequacy of utilizing tools. If we assume that people do not follow the law of the excluded middle, we can use intuitionistic logic. If we believe that additional conditions may invalidate the conclusions drawn earlier, we use one of the non-monotonic logics. Our beliefs often contain so-called propositional attitudes, i.e. phrases indicating cognitive attitude of a subject to the given proposition. They are elements of the class of phrases called modalities. Formal calculi that deal with modalities are modal logics. There are deontic modal logic used to capture reasoning about moral permissibility and obligation, alethic modal logic for reasoning about possibility and necessity *etc.* Shortly speaking, the variety of logics is very great. Having different logics is helpful as each of them captures complementary aspect of the structure of thinking and reasoning. Even though the application of a particular logic to model specific resoning is well justified, it is still vulnerable to the accusation of being inadequate.

3.3. Logical calculi that allow contradictions

Classical logic (as well as most non-classical ones) does not permit belief system to be inconsistent. This kind of calculi do not distinguish between inconsistency and triviality of the system. This means that systems developed within this formal framework are trivial if and only if they are inconsistent. Nevertheless, there is a whole group of logics that allows contradictions within the body of beliefs. The logical calculus that copes with inconsistent yet non-trivial systems is paraconsistent logic.

In a broad sense paraconsistent logic is built by limiting the scope of the law of contradiction:

$$\neg(p\wedge\neg p).$$

Paraconsistent logic has been developed since the beginning of the twentieth century. One of its precursors was Jan Łukasiewicz. In the monograph *On the Principle of Contradiction in Aristotle* (Łukasiewicz, 1971) he distinguishes between three kinds of this principle: logical, metaphysical and psychological one. The latter reads:

> No one can believe that the same thing can (at the same time) be and not be. (Aristotle, n.d., G 3, 1005 b 23-26)

> Two acts of believing which correspond to two contradictory propositions cannot obtain in the same consciousness. (Łukasiewicz, 1971, p. 488)

For our discussion, it is important that according to Łukasiewicz psychological law of contradiction does not hold. The experience of our everyday life indicates that in certain situations people can hold contradictory beliefs.

Chronologically, the first paraconsistent calculus was the discursive logic created by Stanislaw Jaśkowski. (His motivation was to create the logic suitable to formalize the discussion, which often contains contradictory claims of the opponents). However, paraconsistent logic gained its reputation only in the 1950s, thanks to the works of Newton da Costa. Since then, it has been rapidly developed. There are several different paraconsistent calculi. Probably, the most popular today is relevant logic of Graham Priest and Richard Routley.

3.4. Logical calculi are “static”

All logical calculi mentioned above are static and are unable to capture the dynamics of the acquisition and rejection of beliefs. Let us return to the example with an individual holding the following three beliefs:

(1*) $\forall x(B(x) \rightarrow S(x))$,
(2*) $B(a)$,
(3*) $\neg S(a)$.

Suppose that she realized that her belief system contains a pair of contradictory propositions – “Jeremiah is a sophisticated” and its negation. What can she do in this situation? Of course, she may claim that her belief system is paraconsistent, but let us assume additionally that she does not feel good with contradictions. In that case, she may simply reject belief (3). She may also discard belief (2) and state that Jeremiah is not a bullforg, but – for example – a green tree frog. Finally, she may reject the general claim about the sophistication of bullfrog species (1) or state that Jeremiah is so unique that the general rule should take the form of:

(1**) $\forall x(B(x) \wedge x \neq a \rightarrow S(x))$, None of the logical calculi can indicate, which way out of the ones mentioned above is the best[1].

3.5. The problem of assessing the degree of beliefs and ordering the choices

Similar problems beset all attempts to formalize thinking and reasoning in terms of probability theory and the rational choice theory. Moreover, in the case of these formalisms, there are other issues that I will only signal here. One of them is the fact that it is not possible to assess the strength with which one holds a belief. It is hard to even introspectively determine the exact degree of one's own beliefs. The degree of a belief probably varies depending on the situation in which one find herself, the time, the amount of information she has and the context. Similarly, it is hard to assess all the benefits that our choices can bring, thus organize them in one scale. Furthermore, when all given options can bring equal benefit, the criterion of rational choice theory is not applicable.

[1] However, there is some formal tool that can be helpful in these situations – it is Belief Revision Theory, sometimes called AGM, because of its authors Carlos Alchourrón, Peter Gärdenfors and David Makinson. The main principle of AGM is the minimal change principle. It means that the change in belief system shall lead to the loss of as few previous beliefs as possible. Anyway, AGM is not a logical calculus *sensu stricto*.

3.6. Logical omniscience

The last issue I would like to address is the problem of deductive closure of the set of beliefs. In the set of beliefs contradiction can be hidden – a pair of contradictory propositions may not occur until a proper deduction is carried out. It seems that the claim that the set of beliefs is closed under the operations of logical consequence is too strong requirement for cognitive systems, at least for the human mind. For this reason, that kind of problems are often called the problems of logical omniscience.

Let us recall once again the toy example with sophisticated Jeremiah. In this case the inconsistency in the set of beliefs occurs indirectly. There is no doubt that the subject may be unintentionally, that is completely and absolutely, unaware of an implication which ultimately results in a contradiction withe others belief held. We are not logically omniscient – in the sense that we cannot immediately deduce all consequences that can be derived from a set of propositions. Anyone who demands such ability is clearly asking too much (da Costa, French, 1990, p. 185).

First of all, there are many trivial consequences of a given set of beliefs which would simply clutter up one’s mind if added to the set of beliefs held *explicite*[2]. Of course, whether an im-

[2] Assuming that one can by unaware of a belief held *explicite*, as Harman does (Harman, 1986, p. 14)

plied belief is deemed trivial or not depends on the context, but even non trivial implications might escape the human mind, since we are not omniscient even "locally" (da Costa, French, 1990, p. 185).

Sometimes it takes a complex and lengthy proof for one to become aware of certain implication of her belief system. Shortly speaking, the implications may not be obvious in any sense. It seems that such situations occur quite frequently. There are many examples of that, e.g., from the history of science, where certain logical consequences of a hypothesis where not perceived at given time and its later discovery contributed to its confirmation.

Of course, we can determine whether given implication is obvious or not in terms of length of the proof needed to derive it and the number of beliefs involved. Hence, one can say that success in detection of contradiction arisen from the set of belief depends on, i.a., logical competency of cognitive subject. Nevertheless, taking a very strong and unrealistic assumptions concerning logical competency of a cognitive subject it is estimated that consistency test for the very modest set of 138 beliefs would take longer than the current age of the universe (Cherniak, 1984).

4. Conclusions

Despite the above criticisms of normative theories of thinking and reasoning, it is clear that they are not irrelevant for the descriptive theories. NTTR are important for the DTTR for at least three reasons.

Firstly, they provide some theoretical framework needed to operationalize concepts of thinking and reasoning. They also determine what can be expected from thinking subject as well as what constitutes a successfully performed reasoning and what constitutes a failure. For example, if we want to investigate whether people reason deductively, we need first determine what the deduction is and this term belongs to logic and logical theories of reasoning.

Secondly, NTTR often serve as the starting point for the the descriptive theories. There is something more than a simple comparison to the normative theory. Many of them are built directly on the formal approaches, e.g., Johnson-Laird’s theory of mental models.

The third remark concerns the rationality with which people act in the world. It is hard to assign attribute of having mind to someone who moves or generates sentences in a completely random way. According to many philosophical accounts, in such situation one cannot say that such person has intentions, beliefs or goals. In a general sense, NTTR try to provide conditions that must be fulfilled by the action of the subject to be considered rational (cf. Chater, Oaksford, 2012).

References

Aristotle. (n.d.). *Metaphysics*.

Chater, N., & Oaksford, M., 2012. Normative systems: Logic, probability and rational choice. In: K.J. Holyoak, R.G. Morrison (Eds.), *The oxford handbook of thinking and reasoning.* New York: Oxford University Press.

Cherniak, C., 1984. Computational complexity and the universal acceptance of logic. *Journal of Philosophy*, *81*, pp. 739–758.

Christensen, D., 2004. *Putting logic in its place: Formal constraints on rational belief.* New York: Oxford University Press.

da Costa, N., French, S., 1990. Belief, contradiction and the logic of self-deception. *American Philosophical Quarterly*, *3*, pp. 179–197.

Harman, G. (Ed.), 1986. *Change in view. Principles of reasoning.* Cambridge: MIT Press.

Johnson-Laird, P.N. 2010. Mental models and human reasoning. *Proceedings of the National Academy of Sciences*, 107, no 43, pp. 18243–18250.

Johnson-Laird, P.N., 2009. Mental logic, mental models, and simulations of human deductive reasoning. In: R. Sun (Ed.), *The cambridge handbook of computational psychology*. New York: Cambridge University Press.

Lenzen, W., 1978. Recent work in epistemic logic. *Acta Philosophica Fennica*, *30*, pp. 1–219.

Łukasiewicz, J., 1971. On the principle of contradiction in Aristotle. *The Review of Metaphysics*, *24*, pp. 485–509.

Priest, G., 1998. What so bad about contradictions? *The Journal of Philosophy*, *9*, pp. 410–426.

Segerberg, K., 1999. Two traditions in the logic of belief: Bringing them together. In: H. Ohlbach, U. Reyle (Eds.), *Logic, language and reasoning*. Dordrecht: Kluwer Academic Publishers.